

## LETTER

# PCA-Based Detection Algorithm of Moving Target Buried in Clutter in Doppler Frequency Domain

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**SUMMARY** This letter proposes a novel technique for detecting a target signal buried in clutter using principal component analysis (PCA) for pulse-Doppler radar systems. The conventional detection algorithm is based on the fast Fourier transform-constant false alarm rate (FFT-CFAR) approaches. However, the detection task becomes extremely difficult when the Doppler spectrum of the target is completely buried in the spectrum of clutter. To enhance the detection probability in the above situations, the proposed method employs the PCA algorithm, which decomposes the target and clutter signals into uncorrelated components. The performances of the proposed method and the conventional FFT-CFAR based detection method are evaluated in terms of the receiver operating characteristics (ROC) for various signal-to-clutter ratio (SCR) cases. The results of numerical simulations show that the proposed method significantly enhances the detection probability compared with that obtained using the conventional FFT-CFAR method, especially for lower SCR situations.

**key words:** moving target detection, pulse Doppler radar, principal component analysis (PCA)

## 1. Introduction

Doppler radar is a useful tool for detecting moving targets buried in clutter at the same range gate, and is applicable to air traffic control (ATC) systems [1]. These radars can extract extremely small target echoes from clutter or other components using their Doppler frequency differences. The typical Doppler radar algorithms determine the relative velocity of the target using the FFT, whereas the distance of the target is measured by the time delay of the target echoes [2]. In general, there are two basic forms of Doppler radars: frequency modulated continuous wave (CW) radar systems and pulse-Doppler (PD) radar systems [3]. This study deals with the PD radar systems. PD radar, in general, employs CFAR processing which detects the eminent components from the Doppler spectrum of the received signal compared with those from clutter [3]. Where the velocity of the target is faster than that of the clutter, this method accurately separates the target signal from clutter by determining an appropriate threshold [1], [4]. However, Doppler radars often encounter the severe situation in which the Doppler frequency of the target is completely buried in that of the clutter. Particularly, for long range PD radar systems where the pulse

repetition frequency (PRF) has an upper limit to avoid range ambiguity, the Doppler spectrum of the moving target is often buried in that of the clutter due to frequency aliasing [1], [2]. In such a case, the detection of the target with the conventional FFT-CFAR method becomes extremely difficult.

To overcome the above problem, this letter presents a novel algorithm for target detection based on PCA, which decomposes the observed Doppler spectrum into uncorrelated components [5], [6]. Since the target component, in general, takes a higher singular value (SV), PCA can suppress clutter by removing the lower SVs. In addition, to determine the principal component (PC) of the target, the proposed method employs two evaluation criteria. The first criterion is based on the comparison of the SVs of the PCs and the second introduces a sinusoidal signal detection value, which measures the degree of energy concentration of the reconstructed signals in the frequency domain. The performance evaluation of the proposed method is investigated against the conventional FFT-CFAR method for various SCR cases, using the ROC evaluation, which denotes the target detection probability against false alarm probability in this study. The results from numerical simulations verify that the proposed algorithm substantially enhances the detection probability, especially in the lower SCR situations, compared to the conventional FFT-CFAR method.

## 2. System and Signal Model

Figure 1 illustrates the schematic diagram of a PD radar. We assume a single target in this letter. A number of pulses are transmitted after sinusoidal wave modulation. A target echo is received with a time delay and its Doppler frequency is calculated by the received pulses at the same range, which are sampled by the pulse repetition interval (PRI). This study assumes that the radar beamwidth is adequately wider than the size of target, and that the velocity of the target is regarded as constant in the data acquisition interval. These assumptions are not impractical for general ATC systems, because ATC radar targets a civilian airplane that has gradual acceleration and is more than 100 km away. Thus, in such a far field case, the radial velocity change in the observation interval, usually less than 1 second, can be regarded as negligible. Under this assumption, the received signal with the Doppler frequency  $f_d$  can be expressed as

$$s(n) = A \exp(j2\pi f_d n T_{\text{pri}}), \quad (1)$$

where  $T_{\text{pri}}$  denotes PRI,  $A$  is the amplitude of target echo and

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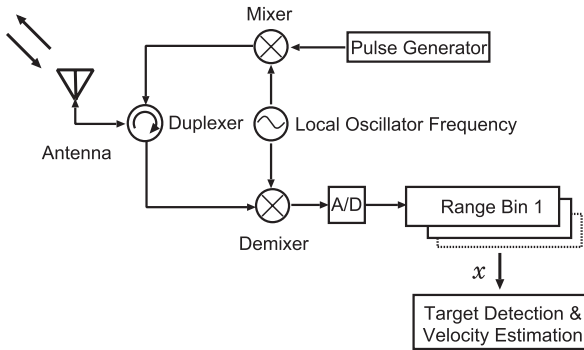


Fig. 1 Schematic illustration of a PD radar system.

$n (= 1, 2, \dots)$  corresponds to the number of pulses. Here, we assume a moving clutter, such as clouds, which are typically found in ATC systems. The clutter signal is formulated using a moving average of signals with identical independent distributions (i.i.d) [2] as in

$$c(n) = \sum_{l=0}^{L-1} \exp \left\{ \frac{-(l - \frac{L}{2})^2}{2\sigma^2} + j2\pi f_c l \right\} \{e_{re}(n-l) + je_{im}(n-l)\}, \quad (2)$$

where  $e_{re}(n)$  and  $e_{im}(n)$  have a zero mean uniform distribution,  $f_c$  and  $L$  denote the Doppler center frequency of  $c(n)$  and the length of a moving window, respectively.  $\sigma$  is the parameter which determines the slope of the window. Note that, Eq. (2) defines the spectrum density of the clutter as the Gaussian distribution, whose spread is dominated by  $\sigma$  and  $L$ . For simplicity, we ignore the thermal noise at the receiver. The observed signal  $x(n)$  is then given by

$$x(n) = s(n) + c(n). \quad (3)$$

### 3. Conventional Method

In general, PD radars detect the target by Doppler spectrum analysis of the received signal. In conventional detection methods, FFT is employed to determine the Doppler spectrum of the target and clutter at a fixed range. The CFAR method is then applied to the output of the FFT to detect the target [3], and the existing CFAR method based on window sliding is introduced [4]. In practice, the CFAR method identifies the target signal by comparing the intensity of the Doppler frequency spectrum with a previously determined threshold  $T(n)$  as

$$T(n) = V_{th} \times \frac{1}{N_w} \sum_{i=-N_w/2}^{N_w/2} |X(n+i)|, \quad (4)$$

where  $X(n)$  denotes the discrete Fourier transform of  $x(n)$ ,  $V_{th}$  is a scaling factor used to adjust the false alarm probability and  $N_w$  denotes the length of the sliding window. Here, the component of the Doppler spectrum that exceeds the CFAR threshold level, i.e.,  $|X(n)| > T(n)$  is regarded as the target component. The FFT-CFAR method can detect

target components accurately when the Doppler frequencies of the target and clutter are sufficiently separated. However, its detection ability immediately decreases when the target is buried inside clutter.

## 4. Proposed Method

To resolve the problem described above, this proposes a target detection algorithm based on PCA. PCA decomposes the observed signals into uncorrelated PCs. Furthermore, to enhance the accuracy of the target detection, the top SV is compared with other SVs, and then an evaluation value specifying the sinusoidal signal detection is introduced in this method.

### 4.1 Target Detection with PCA

PCA is one of the blind source separation techniques and has been used for the suppression of ocean clutter in ground-wave radars [6] or land mine detection in Ground Penetrating Radars(GPR) [7]. After PCA, if the SVs originating from the noise are comparatively lower, the desired signals can be extracted by taking higher SVs. To obtain multiple observed signals, the observed signal matrix  $\mathbf{X}$  is created with a time delay as,

$$\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]^T, \quad (5)$$

where  $\mathbf{x}_m = [x_m(m), x_m(1+m), \dots, x_m(N-1+m)]$  and index  $m$  denotes the channel number while  $N$  represents the total data length. Here, PCA is performed using singular value decomposition (SVD) of the observed signal  $\mathbf{X}$ . Basically, clutter components that have relatively lower SVs compared with those of a target, are eliminated by the PCA compression. The reconstruction signal matrix  $\mathbf{Y}$  after PCA is formulated as

$$\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_P]^T = \mathbf{M}\mathbf{U}^H\mathbf{X} = \mathbf{M}\mathbf{D}\mathbf{V}^H, \quad (6)$$

$$\mathbf{D} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_P, \dots, \sigma_M), \quad (7)$$

where  $^H$  denotes Hermitian conjugate,  $\mathbf{U}$  and  $\mathbf{V}$  are the orthogonal basis matrices, where each column of  $\mathbf{U}$  and  $\mathbf{V}$  consists of the left and right singular vectors of  $\mathbf{X}$ .  $\sigma_i$  denotes a singular value of  $\mathbf{X}$ , and expresses each amplitude of the decomposed signals, where  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_M$  hold. The singular values and vectors correspond to the square roots of the eigenvalues and eigenvectors of  $\mathbf{X}\mathbf{X}^H$  or  $\mathbf{X}^H\mathbf{X}$ , respectively. Here,  $P$  is the number of dominant PCs, which have distinct SVs compared with other SVs, and  $\mathbf{M}$  is the  $P \times M$  rectangular diagonal matrix, where all diagonal elements are one.

To select the desired signal assumed to be a sinusoidal waveform, the proposed method sequentially employs two evaluation values for target detection. First, the evaluation value based on SVs is introduced as

$$\sigma_{\max} = \frac{\max_i \sigma_i}{\sum_{i=1}^P \sigma_i}. \quad (8)$$

$\sigma_{\max}$  measures the predominant ratio of the top SV to other SVs, which typically takes a higher value when the target is present. If  $\sigma_{\max} > T_0$  is satisfied, where  $T_0$  is empirically determined, the next evaluation value is applied for  $\mathbf{y}_{\max}$  corresponding to SV  $\sigma_{\max}$  as

$$e(\mathbf{y}_{\max}) = \frac{\max_n |\mathcal{F}[\mathbf{y}_{\max}](n)|^2}{\sum_{n=1}^N |\mathcal{F}[\mathbf{y}_{\max}](n)|^2}, \quad (9)$$

where  $\mathcal{F}$  denotes the discrete Fourier transform. This evaluation value denotes the energy concentration ratio in the frequency domain. Obviously,  $0 \leq e(\mathbf{y}_{\max}) \leq 1$  holds. Here, if  $\mathbf{y}_{\max}$  forms a sinusoidal wave,  $e(\mathbf{y}_{\max})$  is close to 1, where clutter components are considerably suppressed. Finally,  $\mathbf{y}_{\max}$  is declared as the target, if the condition  $e(\mathbf{y}_{\max}) > V_0$  is also satisfied, where  $V_0$  is also an empirically determined threshold.

### 4.2 Procedure of the Proposed Method

This section presents the actual procedure of the proposed algorithm. Figure 2 illustrates the flow diagram of the proposed method.

Step 1). The observed signal matrix  $\mathbf{X}$  with time delay of the data  $x(n)$  is created in Eq. (5).

Step 2). After applying PCA to  $\mathbf{X}$ , the reconstructed signal  $\mathbf{Y}$  is obtained, and  $\sigma_{\max}$  is determined in Eq. (8).

Step 3). If the following condition is satisfied,

$$\sigma_{\max} > T_0, \quad (10)$$

where  $T_0$  is empirically determined,  $\mathbf{y}_{\max}$  corresponding to  $\sigma_{\max}$  is selected as the tentative target signal, and move on Step 4). Otherwise, it is regarded as no target case.

Step 4). Finally, if the following condition based on Eq. (9) is satisfied,

$$e(\mathbf{y}_{\max}) > V_0, \quad (11)$$

where  $V_0$  is determined empirically,  $\mathbf{y}_{\max}$  is selected as the target signal. Otherwise, it is regarded as no target case.

The proposed method suppresses the substantial power

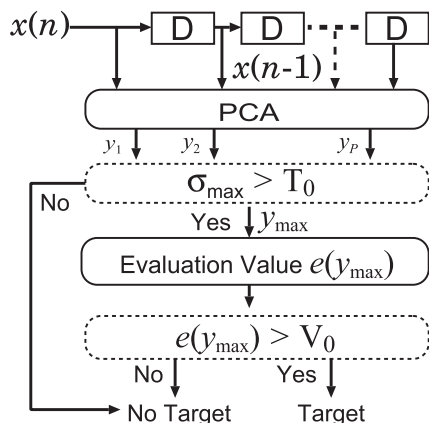


Fig. 2 Flow diagram of the proposed method (D: Time Delay( $T_{pri}$ )).

of the clutter in the reconstructed signal  $\mathbf{Y}$  using PCA decomposition. Furthermore, Step 4) removes a false target signal, which does not have a sinusoidal waveform but has a significantly high SV.

### 5. Performance Evaluation

This section presents the performance evaluation of the proposed method and the conventional FFT-CFAR method using numerical simulation. Here,  $\sigma = 0.08L$  and  $L = 26$  are used in Eq. (2), and the normalized spectral spread of  $c(n)$  is around 0.07, which is typical of weather clutters frequently observed by long range radar systems [8]. Considering that the performance is dominated by  $|f_c - f_d|$ , we set  $f_c = f_d = 0.1$  in which the target signal is completely buried in the clutter spectrum owing to Doppler frequency aliasing caused by a longer PRI. This is the most severe situation in a constant SCR.  $M = 200$ ,  $N = 200$  are used in the proposed method. The performance is evaluated employing ROC curves in terms of target detection probability  $P_d$  against false alarm probability  $P_{fa}$  for a constant SCR. The ROC evaluation is commonly employed to assess a statistical characteristic for a target detection problem [9], [10]. The SCR is defined by

$$SCR = 10 \log_{10} \frac{|A|^2}{E[|c(n)|^2]}, \quad (12)$$

where  $A$  is the amplitude of the target and  $E[*]$  denotes an ensemble averaging. Here, the SCR is averaged by the total trial number used in the numerical simulation.

The upper and lower sides of Fig. 3 show the example of the Doppler spectra of the target signal  $s(n)$  and the clutter

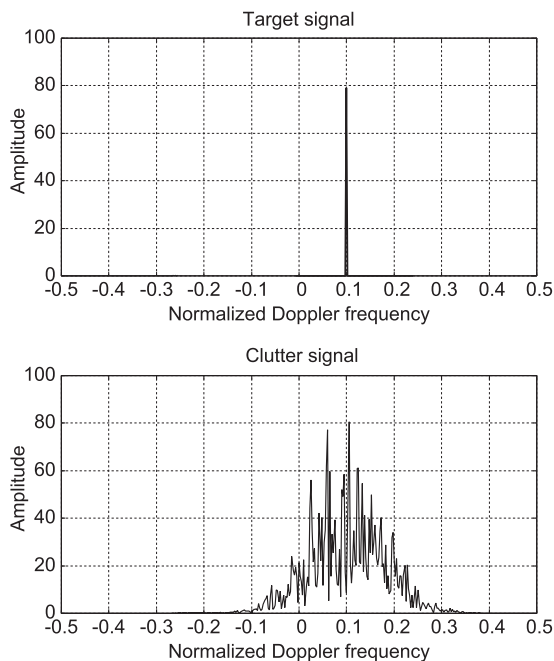
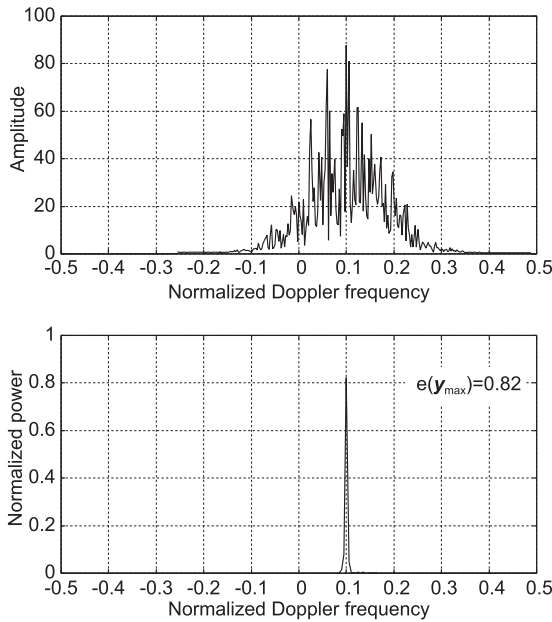
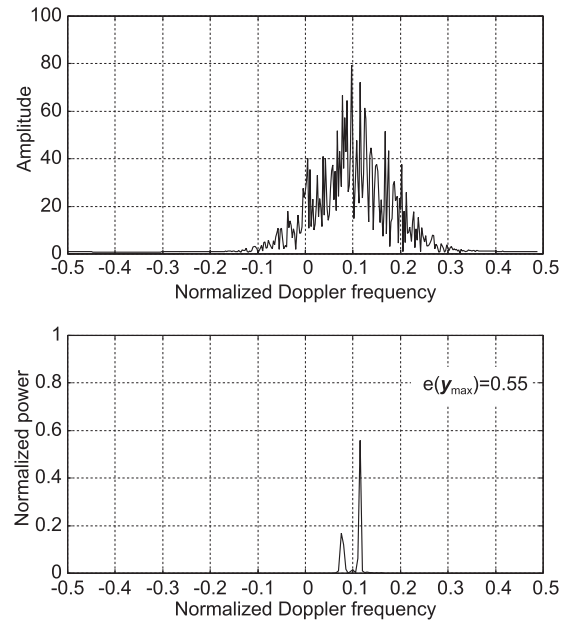


Fig. 3 Example of the Doppler spectra of target  $s(n)$  (upper) and clutter  $c(n)$  (lower) signals at SCR = -10 dB.



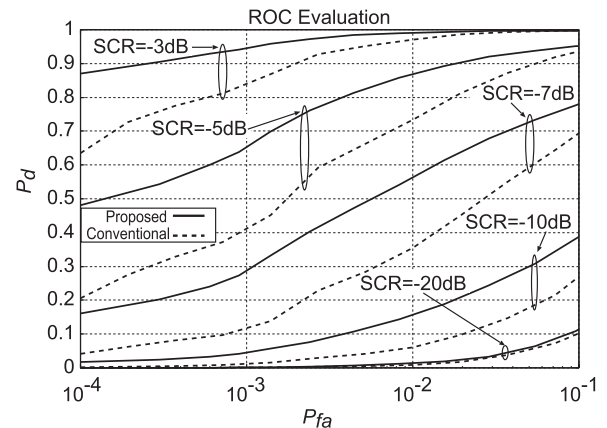
**Fig. 4** Example of the Doppler spectrum of observed signal  $x(n)$  (upper) and the normalized power spectrum of  $\mathbf{y}_{\max}$  (lower) at SCR = -10 dB, in the case that target exists.



**Fig. 5** Example of the Doppler spectrum of observed signal  $x(n)$  (upper) and the normalized power spectrum of  $\mathbf{y}_{\max}$  (lower), in the case that no target exists.

signal  $c(n)$ , respectively, in the case that a target exists. In this case, the maximum amplitudes of target and clutter signals are at the same level, and SCR denotes -10 dB. Moreover, the upper and lower sides of Fig. 4 show the Doppler spectrum of the observed signal  $x(n) = s(n) + c(n)$ , and the normalized power spectrum of the reconstructed signal expressed as  $|\mathcal{F}[\mathbf{y}_{\max}](n)|^2 / \sum_{n=1}^N |\mathcal{F}[\mathbf{y}_{\max}](n)|^2$ , respectively, where  $\mathbf{y}_{\max}$  is the output of the PCA, which has a maximum singular value, and is detected in Step 3) in the proposed method. As shown in the upper side of Fig. 4, the desired target signal, located at  $f_d = 0.1$ , is almost buried in the observed spectrum because of the phase discrepancy between the target and clutter signals at  $f_d = 0.1$ . However, the reconstructed Doppler spectrum has an impulse distribution around the actual Doppler frequency of the target as  $f_d = 0.1$ , and  $e(\mathbf{y}_{\max}) = 0.82$  holds. On the contrary, the upper and lower sides of Fig. 5 present the same view as shown in Fig. 4, in the case that no target exists. Although  $\mathbf{y}_{\max}$  also satisfies Eq. (10) in this case, its Doppler spectrum is broadened, and  $e(\mathbf{y}_{\max})$  holds for the lower value of 0.55. As shown in these examples, a distinct discrepancy can be seen for  $e(\mathbf{y}_{\max})$  between the target present and the no target cases, despite  $\mathbf{y}_{\max}$  for both cases satisfying the condition of the Step 3) in the proposed method. Thus, by introducing the second evaluation as in Eq. (11) in Step 4), only the sinusoidal signal can be accurately extracted as a target signal.

For the statistical evaluation for detection performance, the comparison of ROC curves is shown in Fig. 6 for SCR = -20 dB, -10 dB, -7 dB, -5 dB, and -3 dB. In this case, 10000 clutter scenes are investigated with a constant SCR, where the false alarm rate  $P_{fa}$  is adjusted from  $10^{-4}$  to  $10^{-1}$ , by decreasing each threshold  $V_{th}$  of the conven-



**Fig. 6** Target detection probability against false alarm probability for various SCR cases.

tional and  $T_0$  of the proposed methods, respectively. Here,  $V_0 = 0.85$  is set in the proposed method. It shows that the proposed method enhances the target detection probability compared with the conventional FFT-CFAR method. For example, at SCR = -7 dB, at  $P_{fa} = 10^{-3}$ , the proposed method has  $P_d = 0.29$  whereas the conventional FFT-CFAR method attains  $P_d = 0.11$ . Similarly, for SCR = -5 dB at  $P_{fa} = 10^{-3}$ , the proposed method enhances  $P_d$  to 0.65 compared to  $P_d = 0.41$  with conventional method. Furthermore, this figure quantitatively shows that the proposed method enhances the detection probability in all SCR cases. This excellent result verifies the effectiveness of the PCA compression and the appropriate target detection criteria of the proposed scheme. Furthermore, it should be noted that the ROC of the proposed method is not significantly sensitive

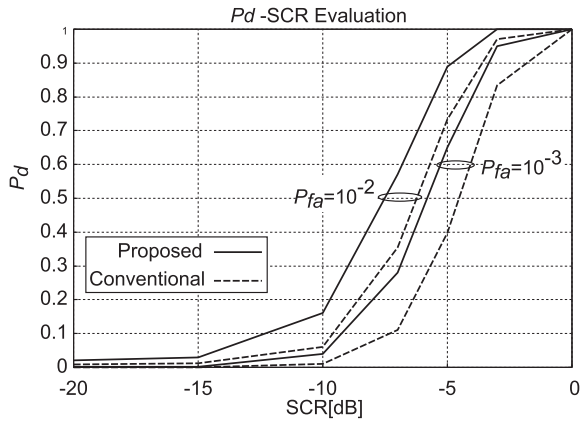


Fig. 7 Target detection probability against SCR at constant  $P_{fa} = 10^{-3}$  and  $P_{fa} = 10^{-2}$ .

to the parameters  $V_0$ ,  $M$  and  $N$ . It has been established that the proposed method has a sufficient ROC improvement over the conventional method, in the cases of  $V_0 = 0.80$  or  $V_0 = 0.90$ , and in also other cases of  $(M, N) = (300, 100)$  or  $(M, N) = (100, 300)$ . In addition, Fig. 7 presents the relationship between  $P_d$  and SCR at constant  $P_{fa} = 10^{-3}$  and  $P_{fa} = 10^{-2}$ . This figure also proves that the proposed method achieves a higher  $P_d$  compared with that attained using the conventional FFT-CFAR method for all SCR cases. Note that, when the effective window length in Eq. (2) increases, the spread of the Doppler spectrum of the clutter becomes narrower around the Doppler frequency of the target. Thus, in this case, both the conventional and proposed methods are barely able to detect the target signal because the effective SCR has become considerably lower. In the future, we plan to enhance the ROC characteristic for these more severe situations.

As a final remark, in realistic situations, there are possibly some fluctuations of the actual Doppler spectrum of the target owing to the small radial velocity variance caused by a gradual acceleration of the target or slight discrepancy in the angle of arrival within the observation interval. It can then be predicted that this kind of Doppler broadening may result in the performance degradation of the proposed method because it is based on the assessment of the impulsive distribution of the Doppler spectrum for target detection. This problem will be also examined in our future studies.

## 6. Conclusions

In this letter, we propose a novel approach for the detection of a moving target in PD radar systems. In contrast to the existing approaches, we focus on target detection when the Doppler spectrum of the target is buried in that of the clutter. The comparison of the ROC curves has shown that the proposed method based on PCA can enhance target detection probability compared with the conventional FFT-CFAR method. The main advantage of the proposed method is that it can detect a target for all SCR cases more efficiently than the conventional FFT-CFAR method. In our future studies, we plan to confirm the effectiveness of the proposed scheme in an experiment.

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